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About magnetic relaxation of partially penetrated screening current in superconductors with various models of flux creep

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Abstract. Magnetic relaxation of the partially penetrated screening current during flux creep has been studied. A theoretical analysis was based on the macroscopic description of the flux creep using the power and exponential equations for current-voltage characteristics of superconductors. The analytical flux-creep solutions were written and compared with numerical simulations of corresponding problems. Equations describing the relaxation of the electromagnetic field, magnetic moment, moving penetration boundary are derived. It is shown that peculiarities of relaxation phenomena are determined by dynamics of the electric field on the surface of the superconductor. The performed analysis allows to formulate nontrivial conjugation conditions that take place on moving boundary of screening current. In accordance with these conditions the electromagnetic field induced inside a superconductor by external perturbations smoothly approaches its undisturbed values. The essential role of the low electric field area of current-voltage characteristics in the flux relaxation and primarily in high-temperature superconductors is shown.

PACS. 74.60.Ge Flux pinning, flux creep, and flux-line lattice dynamics – 74.60.Jg Critical currents – 74.25.Ha Magnetic properties

1 Introduction

The study of the magnetic relaxation of superconductors is very interesting from an application as well as basic-science point of view. The investigations of these non-equilibrium phenomena in the macroscopic approximation become a useful tool for understanding the microscopic mechanisms of pinning. That is why many authors have given considerable attention to the study of magnetic relaxation problems both experimentally and theoretically. However, only the fully penetrated state is well understood. The numerous previous investigations for the cases of partial flux penetration based on the numerical methods did not give full description of magnetic relaxation problem. It should also be noted that the existing solutions for the relaxation problem of the partially penetrated screening current in the high-temperature superconductor with logarithmic current dependence of the potential barrier are ambiguous [1–3]. In the present paper the analytical solutions for the Maxwell equations describing the magnetic relaxation of low- and high-temperature superconductors during flux creep are written. They make it possible to formulate for the first time some physical features of the magnetic relaxation of partially penetrated screening current during flux creep in terms of different phenomenological equations of current-voltage (I-V) characteristics.

2 Simple theoretical models of current-voltage characteristics in the flux-creep regime

The dependence of the electric field on the current induced inside the superconductor by the varying external magnetic field or by the transport current has an essentially nonlinear form. The nonlinear part of current-voltage characteristics is due to many reasons. Numerous studies (see, for example, [4–10] and references cited therein) show that the following equations

$$E = E_C(J/J_C)^n \tag{1}$$
$$E = E_C \exp[(J - J_C)/J_\delta] \tag{2}$$

can be used for the description of I-V characteristics of both the low- and high-temperature superconductors. Here, J_C is the current density at $E = E_C$; n is the creep exponent of the current-voltage characteristic; J_δ is the creep current density.

In equation (2) the electric field is $E_0 = E_C \exp(-J_C/J_\delta)$ at $J = 0$. To avoid this uncertainty, let

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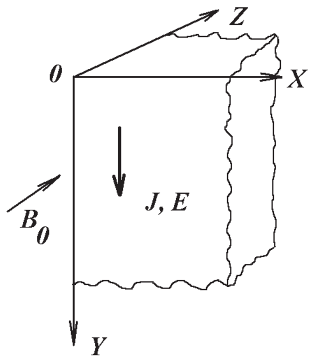

Fig. 1. Superconductor in external magnetic field $B_0(t)$.

us write equation (2) as

$$E = E_C \exp[(J - J_C)/J_\delta] - E_0. \tag{3}$$

Equation (1) corresponds to a logarithmic current dependence of the potential barrier [1,2] when the flux creep is determined by numerous spatial defects of the superconductor. The thermally activated model [4] with linear current dependence of the potential barrier lies at the basis of the exponential relation (2). This model describes flux-creep state of the superconductor with point defects of its structure. There are also some macroscopic reasons leading to an exponential rise of the I-V characteristic. In particular, it may result from the bulk heterogeneity of superconducting properties inside the sample. In addition to the bulk heterogeneity of critical parameters the superconductor may have the longitudinal heterogeneity. However, the $E(J)$ relations of such superconductors are also approximated satisfactorily by equation (1).

3 Model

Let us consider an ideally cooled half-infinite superconductor (Fig. 1) placed in the homogeneous external magnetic field $H_0 (H_0 \gg H_{c1}, H_{c1}$ is the lower critical field) parallel to its surface in the Z-direction when this field induces a uniform induction $B_0 = \mu_0 H_0$. Assume that any field disturbance, which is much smaller then B_0, takes the constant value $B_{0,i}$ at $t = t_i$ and it does not change the constants J_C, J_δ, n. This perturbation produces the extra magnetic induction and induces the screening current in the Y-direction that diffuses into the superconductor. For this case the Maxwell equations describing the macroscopic decay of the induced electric field $E = E_y(x,t)$ are given by one-dimensional unsteady equations

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \begin{cases} \dfrac{J_C}{nE}\left(\dfrac{E}{E_C}\right)^{1/n}\dfrac{\partial E}{\partial t}, & (4)\\[2ex] \dfrac{J_\delta}{E+E_0}\dfrac{\partial E}{\partial t}, & (5)\end{cases}$$

which satisfy the boundary conditions

$$\frac{\partial E}{\partial x}(0,t) = 0, \quad E(\infty,t) = 0 \tag{6}$$

and initial condition $E(x,t_i) = E_0(x)$. Along with these conditions an additional relation

$$\mu_0 \int_0^\infty J(x,t)\mathrm{d}x = B_{0,i} \quad t \geq t_i \tag{7}$$

is needed, which follows from the conservation law of induced screening current. As will be shown below, it plays an important role in the correct description of the flux-creep states.

It should be noted that the penetration depth of the electric field is not known *a priori*. Therefore, conditions (6, 7) are based on the assumption that the screening current flows over the total volume of superconductor like in the computer simulation models (see, for example, [11,12]).

Different specimens are investigated in experiments. To study the electromagnetic properties of superconductors of arbitrary shape general calculation algorithms were presented in [13,14]. However, in many practical cases two- or even one-dimensional models may be used [15]. Moreover, 1D consideration allows one to understand the physical features of this phenomenon solving simplified equations without large volume of computations. From this point of view the possible 1D solution in closed form is convenient to evaluate the experiments even for realistic geometry of superconductors. So, let us write the analytical solutions of the problem under consideration using the method of the scaling functions and discuss the peculiarities of electromagnetic states of superconductors during magnetic relaxation of partially penetrated screening current which are formulated in present theory for the first time.

4 Relaxation phenomena in a superconductor with the power current-voltage characteristic

Let us introduce the dimensionless variables $e = E/E_C, X = x/L_x, \tau = t/t_x$, where $L_x = B_{0,i}/\mu_0 J_C$, $t_x = B_{0,i}^2/(\mu_0 J_C E_C)$. The scaling solution of the problem (4, 6, 7) can be found in the form

$$e = (\tau + \tau_0)^q W(Z), \; X = (\tau + \tau_0)^p Z,$$
$$q = -n/(n+1), \; p = 1/(n+1)$$

where τ_0 is the constant to be determined. Then the partial differential equation (4) is reduced to an ordinary differential equation

$$(n+1)\frac{\mathrm{d}^2 W}{\mathrm{d}Z^2} + \frac{Z}{n} W^{\frac{1-n}{n}} \frac{\mathrm{d}W}{dZ} + W^{\frac{1}{n}} = 0, \tag{8}$$

with the boundary conditions

$$\frac{\mathrm{d}W}{\mathrm{d}Z}(0) = 0, \quad W(\infty) = 0. \tag{9}$$

Equation (8) with condition (9) has the analytical solution

$$W(Z) = \left[\frac{n-1}{2n(n+1)}\left(Z_0^2 - Z^2\right)\right]^{\frac{n}{n-1}}$$

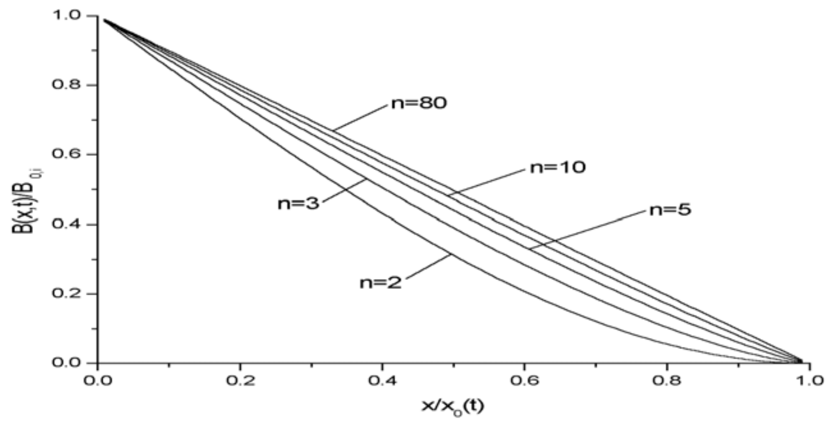

Fig. 2. Scaling distribution of the magnetic induction in a superconductor with the power I-V characteristic.

where Z_0 is an unknown constant. According to (7) the value Z_0 is given by

$$\int_0^{z_0} W^{\frac{1}{n}} \mathrm{d}y = 1. \tag{10}$$

Then it can be written as

$$Z_0 = \left[\frac{2n(n+1)}{n-1}\right]^{\frac{1}{n+1}} \left(\frac{1}{\Psi_1}\right)^{\frac{n-1}{n+1}}, \;\; \Psi_1 = \int_0^1 (1-y^2)^{\frac{1}{n-1}} \mathrm{d}y.$$

It is easy to see that both value of scaling function $W(Z)$ and all derivatives over Z vanish at a point $Z = Z_0$. Hence, function $W(Z)$ approaches smoothly to zero value at $Z = Z_0$.

In accordance with scaling solution the decay of the electromagnetic field in the superconductor with the power I-V characteristic is described by the following expressions

$$E(x,t) = E_a(t)\left(1 - \frac{x^2}{x_0^2}\right)^{\frac{n}{n-1}},$$

$$B(x,t) = B_{0,i}\left[1 - \frac{1}{x_0\Psi_1}\int_0^x \left(1 - \frac{y^2}{x_0^2}\right)^{\frac{1}{n-1}} \mathrm{d}y\right]. \tag{11}$$

Here

$$E_a(t) = E_C\left(\frac{t_n}{t+t_0}\right)^{\frac{n}{n+1}}, \quad t_n = \frac{n-1}{2n(n+1)\Psi_1^2}\frac{B_{0,i}^2}{\mu_0 J_C E_C}.$$

In this case, the time-dependent equation of the magnetization boundary is given by

$$x_0(t) = \frac{B_{0,i}}{\mu_0 J_C \Psi_1}\left(\frac{E_C}{E_a(t)}\right)^{\frac{1}{n}}. \tag{12}$$

In the given solution constant t_0 is unknown. To estimate it let us use the value of the electric field $E_{0,i} = E_0(0)$ on the surface of the superconductor at $t = t_i$. This gives $t_0 = t_e - t_i$, where t_e is the characteristic time decay, which should be written as

$$t_e = \frac{n-1}{2n(n+1)\Psi_1^2}\frac{B_{0,i}^2}{\mu_0 E_{0,i}^{(n+1)/n}}\frac{E_C^{1/n}}{J_C}.$$

The curves in Figure 2 show the influence of power I-V characteristic on the distribution of the magnetic induction in the magnetization region. It is easy to see that the value $n = 10$ describes satisfactorily the boundary value above which the distribution of the magnetic induction inside the superconductor is nearly linear. This estimate gives reasons for the use of simplified methods of relaxation analysis as it was done, for example, in [3].

The written for the first time full analytical solution in which all constant are exactly determined shows that magnetic relaxation in a superconductor with the power I-V characteristic is accompanied by formation of a special state. Firstly, the electromagnetic field penetrates as a characteristic wave at the finite rate

$$\frac{\mathrm{d}x_0}{\mathrm{d}t} = \frac{2n\Psi_1}{n-1}\frac{E_a(t)}{B_{0,i}}$$

although there is a stable voltage inside superconductor due to flux creep. Secondly, the conjugation area between perturbed and unperturbed values of the electric field and magnetic induction exists, since the conditions $E = 0, B = 0, \partial^k E/\partial x^k = 0, \partial^k B/\partial x^k = 0, k = 1, 2, 3$ ... take place at the moving boundary $x_0(t)$. As a result, the differential resistivity of superconductor $\rho_d = \partial E/\partial J$ has a special change. According to the formula

$$\rho_d(x,t) = n\frac{E_C}{J_C}\left(\frac{E_a(t)}{E_C}\right)^{\frac{n-1}{n}}\left(1 - \frac{x^2}{x_0^2}\right)$$

it monotonously decreases and $\rho_d \equiv 0$ at $x_0(t) \geq 0$.

The importance of these results should be underlined. They are not *a priori* evident and show the general peculiarity of the relaxation states. These phenomena are determined by electric field dynamics on the surface of the superconductor. Let us reveal that it takes place for all equations of the I-V characteristics of superconductors. Integrating equation $\partial B/\partial t = -\partial E/\partial x$ from 0 to x_0 and using conditions $E(x_0,t) = 0, B(x_0,t) = 0$ one can obtain the time-dependent equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_0^{x_0} B(x,t)\mathrm{d}x = E_a(t)$$

describing the moving boundary evolution as the function of the electric field variation on the surface

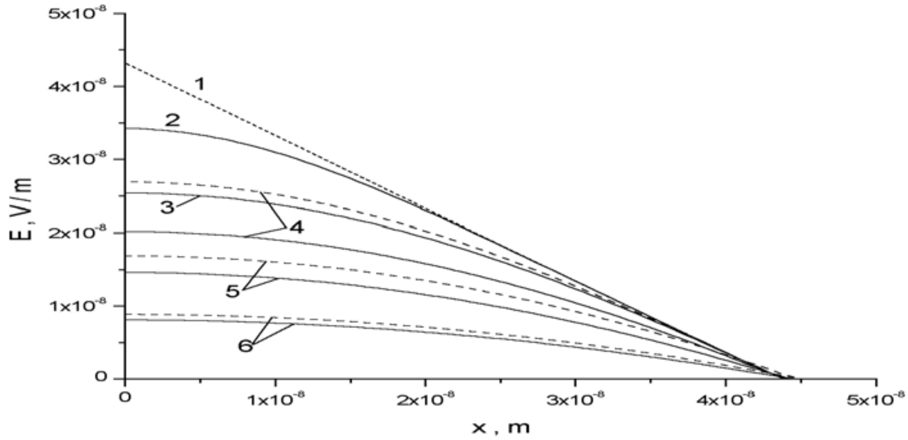

Fig. 3. Distribution of electric field in Nb-Ti superconductor with the power I-V characteristic at relaxation stage (— numerical solution, - - - - scaling solution): $1 - t = 2 \times 10^{-4}$ s, $2 - t = 2.001 \times 10^{-4}$ s, $3 - t = 2.005 \times 10^{-4}$ s, $4 - t = 2.01 \times 10^{-4}$ s, $5 - t = 2.02 \times 10^{-4}$ s, $6 - t = 2.05 \times 10^{-4}$ s.

of superconductor. This law takes place also during ramp rate stage [16].

The written solution allows one to understand the correctness of depiction of the magnetic relaxation in the superconductor with logarithmic dependence of the potential barrier on the current presented in [1,2]. As shown in [3], the formula proposed in [1] to estimate the magnetic moment of the superconducting slab for the partial screening current penetration has error. Moreover, according to the above solution the conditions of the smooth conjugation of the disturbed and undisturbed values of the electromagnetic field at the moving boundary $x_0(t)$ must be satisfied. The same conditions must be also fulfilled for the problem discussed in [1,2]. However, the solution proposed in [1] does not satisfy these conditions. The error made in [1] is due to that the additional condition $J(x,t) = J_C$ *at* $x = 0, t = 0$ was used to determine unknown constant appearing in the solution instead of the condition similar to (7).

Let us use the above-formulated solution to define the magnetic moment

$$\mu_0 M(t) = \frac{1}{a}\int_0^{x_0} B(x,t)\mathrm{d}x - B_{0,i}$$

of the superconducting slab of half thickness a before the full penetration state. According to (11) the result is

$$-\mu_0 M(t)/B_{0,i} = 1 - \varphi_n x_0(t)/a$$

for all $x_0(t) < a$. Here,

$$\varphi_n = 1 - \frac{1}{\Psi_1}\int_0^1 \Psi(\eta)\mathrm{d}\eta, \quad \Psi(\eta) = \int_0^\eta (1-y^2)^{\frac{1}{n-1}}\mathrm{d}y.$$

This formula can be reduced to the relation

$$M(t) = M_i + M_1\left(\frac{E_C}{E_{0,i}}\right)^{\frac{1}{n}}\left[\left(\frac{t-t_i+t_e}{t_e}\right)^{\frac{1}{n+1}} - 1\right]\varphi_n, \quad t \ge t_i$$

where M_i is the magnetic moment of the slab at $t = t_i$ and $M_1 = B_{0,i}^2/(\mu_0^2 J_C a \Psi_1)$.

These expressions show that magnetic moment relaxation of partially penetrated currents depends on history of external magnetic field variation, size of the slab and its physical properties, similarly to the full penetration case [1,17] when the universal electric field distribution $E = f(x)\phi(t)$ takes place after some transient time. In the mean time, unlike the full penetration case the above-written formulae indicate that electric field distribution during incomplete flux penetration has another form, which is written as follows $E = E_a(t)\zeta[x/x_0(t)]$, *i.e.*, the magnetic relaxation is determined by dynamics of the moving boundary $x_0(t)$ and electric field decay $E_a(t)$ on the surface.

Differentiating $M(t)$ over the time it is easy to find

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{M_1\varphi_n}{(n+1)(t-t_i+t_e)}\left(\frac{t-t_i+t_e}{t_n}\right)^{\frac{1}{n+1}}.$$

This formula allows one to find two characteristic magnetic relaxation rates. If $t - t_i \ll t_e$, then the short-time rate of the magnetic moment relaxation is almost constant and can be estimated as

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \frac{M_1\varphi_n}{(n+1)t_e}\left(\frac{E_C}{E_{0,i}}\right)^{\frac{1}{n}} = \frac{2n\Psi_1\varphi_m}{n-1}\frac{E_{0,i}}{\mu_0 a}.$$

If $t - t_i \gg t_e$, then the decreasing long-time relaxation rate is

$$\frac{\mathrm{d}M}{\mathrm{d}t} \approx \frac{M_1\varphi_n}{(n+1)(t-t_i)}\left(\frac{t-t_i}{t_n}\right)^{\frac{1}{n+1}}$$

which in logarithmic time scale can be written as

$$\frac{\mathrm{d}M}{\mathrm{d}\ln t} \approx \frac{M_1\varphi_n}{n+1}\frac{t}{t-t_i}\left(\frac{t-t_i}{t_n}\right)^{\frac{1}{n+1}}.$$

To illustrate the scaling solution possibility the simulation results of corresponding electrodynamic states are shown in Figures 3–5. Figure 3 displays the initial relaxation stage in the low-temperature superconductor ($n = 80$, $E_C = 10^{-4}$ V/m, $J_C = 4 \times 10^9$ A/m^2). The

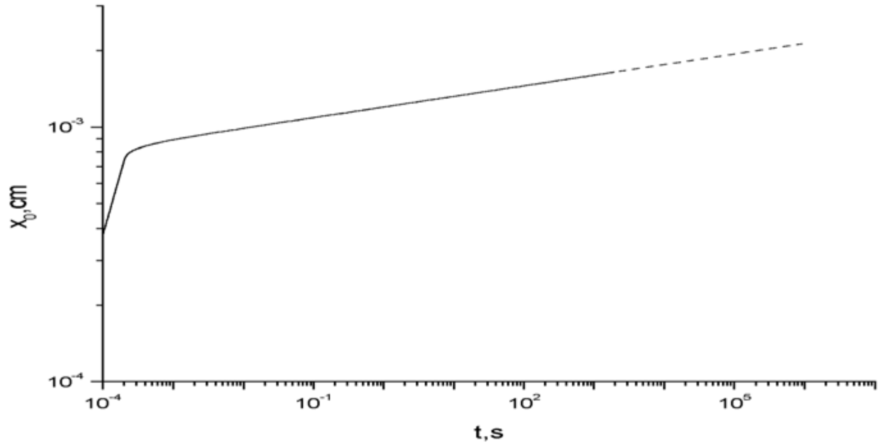

Fig. 4. Dynamics of moving boundary in YBCO superconductor with the power I-V characteristic at ramp rate and relaxation stages: — numerical solution, - - - - - scaling solution.

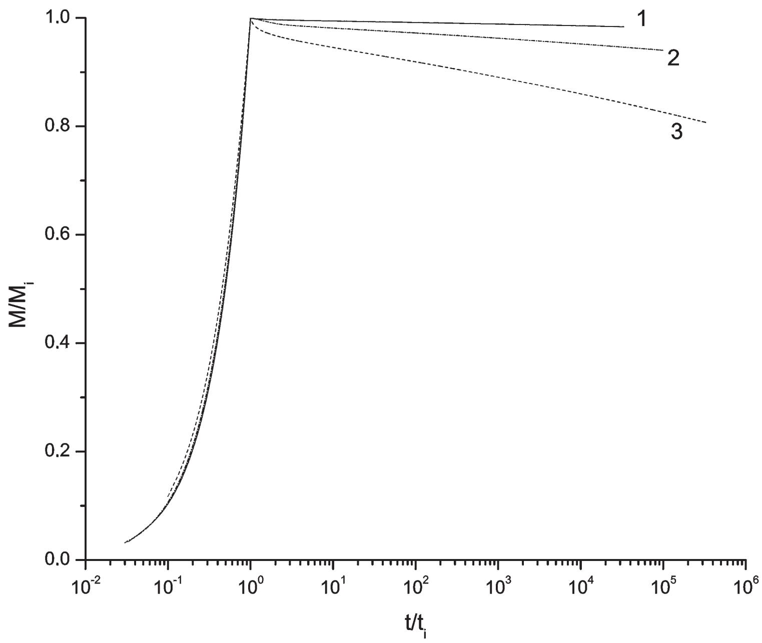

Fig. 5. Time dependence of magnetic moment of YBCO superconductor during ramp rate and relaxation stages: $1 - \mathrm{d}B/\mathrm{d}t = 10^{-4}$ T/s, $t_i = 3 \times 10^{-4}$ s, $2 - \mathrm{d}B/\mathrm{d}t = 10^{-4}$ T/s, $t_i = 10^{-3}$ s, $3 - \mathrm{d}B/\mathrm{d}t = 10^{-3}$ T/s, $t_i = 3 \times 10^{-4}$ s.

electric field distribution $E_0(x)$ specified at $t = t_i$ was taken according to the numerical simulation of the ramp rate stage. Here and below the ramp rate simulations are based on the numerical solution of the equations (4, 5) with conditions

$$\frac{\partial E}{\partial x}(0,t) = -\frac{\mathrm{d}B}{\mathrm{d}t},\ E(x_0,t) = 0,$$

$$E(x,0) = 0,\ \mu_0 \int_0^{x_0} J(x,t)\mathrm{d}x = \frac{\mathrm{d}B}{\mathrm{d}t}t.$$

Curve 1 corresponds to electric field distribution at $t_i = 2 \times 10^{-4}$ s for the case $\mathrm{d}B/\mathrm{d}t = 1$ T/s. Solid curves are obtained using numerical solution of magnetic relaxation problem described by (4, 6, 7). Dashed curves 4–6 are calculated according to the formulae (11, 12). Figure 4 shows the curves describing the evolution of the moving boundary of the magnetization region inside the high-temperature superconductor with parameters defined by relations $n = 23$, $E_C = 1.778 \times 10^{-4}$ V/cm, $J_C = 2 \times 10^5$ A/cm^2. These curves were obtained under the assumption that the external magnetic field was increased at the rate $\mathrm{d}B/\mathrm{d}t = 7.5 \times 10^{-3}$ T/s and fixed at $t \geq 2 \times 10^{-4}$ s.

It is easy to see that scaling solution is the good approximation that can be used to describe exactly enough complete period of the magnetic relaxation. To demonstrate this capability the temporal peculiarities of the magnetic moment evolution of mentioned-above high-temperature superconductor is shown in Figure 5.

5 Relaxation phenomena in a superconductor with the exponential current-voltage characteristic

Let us investigate the magnetic relaxation in the superconductor with I-V characteristic (3). Introducing dimensionless variables $e = E/E_C$, $X = x/L_x$, $\tau = t/t_x$, where $L_x = B_{0,i}/\mu_0 J_\delta$, $t_x = B_{0,i}^2/(\mu_0 J_\delta E_C)$, the unknown electric field distribution can be found in the form of power series expansion using the small parameter $e_0 = E_0/E_C \ll 1$, *i.e.*, $e = \varepsilon_0 + \varepsilon_1 e_0 + \varepsilon_2 e_0^2 + ...$

Confining ourselves to the zero approximation one can reduce the problem (5, 6, 7) to the integration of the following partial differential equation

$$\frac{\partial \varepsilon_0}{\partial \tau} = \varepsilon_0 \frac{\partial^2 \varepsilon_0}{\partial X^2} \tag{13}$$

with the boundary condition

$$\frac{\partial \varepsilon_0}{\partial X}(0, \tau) = 0 \tag{14}$$

and the preservation condition of the induced current with density $j = 1 + \delta \ln(\varepsilon_0 + e_0)$

$$\int_0^{x_0} j(X, \tau) \mathrm{d}X = \delta. \tag{15}$$

Equation (15) is written already in accordance with the existence of the finite length of the magnetization region, at which the condition $\varepsilon_0(X_0, \tau) = 0$ must be fulfilled. Here, $j = J/J_C$, $\delta = J_\delta/J_C$.

Due to the smallness of e_0, the approximations $\varepsilon_1, \varepsilon_2, \varepsilon_3$, ... will have practically no quantitative influence on the value of $e(X, \tau)$. Therefore, ε_0 can be used as the fundamental solution and the main investigated phenomenon occurs on its background.

The problem defined by equations (13–15) is more complex than the one investigated above. Therefore, let us describe its solution in a more detailed way. To find the electric field distribution in scaling form, the corresponding solution should be written as the product of two functions, each of which depends only on one argument

$$\varepsilon_0 = T(\tau)W(Z), \quad Z = X/X_0(\tau).$$

Then equation (13) can be written as

$$\frac{\mathrm{d}T}{\mathrm{d}\tau}W - \frac{\mathrm{d}X_0}{\mathrm{d}\tau}\frac{Z}{X_0}T\frac{\mathrm{d}W}{\mathrm{d}Z} = \frac{T^2}{X_0^2}W\frac{\mathrm{d}^2W}{\mathrm{d}Z^2}. \tag{16}$$

Since the functions $T(\tau)$ and $X_0(\tau)$ are initially taken arbitrarily, let us demand that they would satisfy the equality

$$-\alpha X_0 \mathrm{d}T/\mathrm{d}\tau = T\mathrm{d}X_0/\mathrm{d}\tau$$

where α is the constant that must be determined. This equality allows the connection between $T(\tau)$ and $X_0(\tau)$ to be found. It has the form

$$X_0(\tau) = \gamma T^{-\alpha}(\tau). \tag{17}$$

Here, γ is the constant, which can be calculated.

The given expressions permit the variables in equation (16) to be separated. The corresponding relation has the form

$$\frac{T^2}{X_0^2}\frac{\mathrm{d}T}{\mathrm{d}\tau} = \frac{W\dfrac{\mathrm{d}^2W}{\mathrm{d}Z^2}}{W + \alpha Z\dfrac{\mathrm{d}W}{\mathrm{d}Z}} = -\beta.$$

Here, β is the separation constant. Taking into account equation (17) it is easy to obtain $T(\tau)$ with the accuracy of the unknown constant τ_0

$$T(\tau) = T_1(\tau)/\beta, \quad X_0(\tau) = \gamma_1(\tau + \tau_0)^{\alpha/1+2\alpha}$$

where

$$T_1(\tau) = \frac{\gamma_1^2}{(1+2\alpha)(\tau+\tau_0)^{1/1+2\alpha}},$$
$$\gamma_1 = [\gamma\beta^\alpha(1+2\alpha)^\alpha]^{1/1+2\alpha}.$$

Let us introduce a new function $W_1 = W/\beta$. Then solution being equal to

$$\varepsilon_0 = T_1 W_1 = \frac{\gamma_1^2 W_1}{(1+2\alpha)(\tau+\tau_0)^{1/1+2\alpha}}$$

does not depend on constant β, and the initial problem is reduced to the integration of a differential equation

$$W_1\frac{\mathrm{d}^2W_1}{\mathrm{d}Z^2} + \alpha Z\frac{\mathrm{d}W_1}{\mathrm{d}Z} + W_1 = 0 \tag{18}$$

with the boundary conditions

$$\frac{\mathrm{d}W_1}{\mathrm{d}Z}(0) = 0, \quad W_1(1) = 0.$$

The possible solution for equation (18) has the following character. Firstly, it is easy to show that all the derivatives $\mathrm{d}^kW_1(Z)/\mathrm{d}Z^k, k = 1, 2, 3,$ ... equal zero at $Z = 1$. In addition, two integral equalities follow from equation (18)

$$1 + \alpha\int_0^1 \frac{Z}{W_1}\frac{\mathrm{d}W_1}{\mathrm{d}Z}\mathrm{d}Z = 0,$$
$$\int_0^1 \left(\frac{\mathrm{d}W_1}{\mathrm{d}Z}\right)^2 \mathrm{d}Z + (\alpha - 1)\int_0^1 W_1 \mathrm{d}Z = 0.$$

As, according to its physical sense, $W_1 \geq 0$ and $\mathrm{d}W_1/\mathrm{d}Z \leq 0$, then these equalities exist only when $0 < \alpha < 1$.

Thus, the distribution of the electromagnetic field inside the superconductor with the exponential I-V characteristic can be formally determined within the accuracy of three constants α, γ, τ_0 after integrating equation (18). From the formal point of view, condition (15) should be used to determine α and γ. However, this condition does not allow the final solution in scaling approximation to be found. Physically this conclusion corresponds to breaking the scaling form in the character of the desired solution. At the same time its real evolution can be approximated by the scaling solution asymptotically that permit to estimate the unknown constants α and γ. For this purpose let us simplify I-V characteristic described by equation (3) expanding the exponential term as a power series. Using the linear term of this series let us find α and γ for the following I-V characteristic $J \approx J_C(E/E_0)^\delta \xi$, where $\xi < 1$ is the correction term of the linear expansion. This value can be calculated from condition: $E = E_C$ at $J = J_C$. Then $\xi = (E_0/E_C)^\delta$. For this approximation condition (15) can be expressed as

$$\delta = \gamma_1^{1+2\delta} \frac{(\tau+\tau_0)^{\frac{\alpha-\delta}{1+2\alpha}}}{(1+2\delta)^\delta} \int_0^1 W_1^\delta(Z)\mathrm{d}Z.$$

As its right part should not depend on time then this requirement leads to equalities

$$\alpha = \delta, \quad \gamma_1 = \left[\delta(1+2\delta)^\delta \Big/ \int_0^1 W_1^\delta \mathrm{d}Z\right]^{\frac{1}{1+2\delta}}. \tag{19}$$

As $\delta \ll 1$ then γ_1 can be approximated by the expression

$$\gamma_1 \approx [\delta(1+2\delta)^\delta]^{1/1+2\delta}. \tag{20}$$

To determine $W_1(Z)$, let us transform equation (18) to the equivalent integral equation. After some algebra the result can be written as

$$W_1(Z) = \frac{1-Z^2}{2} - \alpha \int_1^Z \mathrm{d}x \int_0^x \frac{y}{W_1}\frac{\mathrm{d}W_1}{\mathrm{d}y}\mathrm{d}y.$$

Because of α being small the scaling function $W_1(Z)$ is exactly enough approximated by the zero solution, *i.e.*,

$$W_1(Z) \approx (1-Z^2)/2.$$

In accordance with these approximations the relaxation phenomena in the superconductor with the exponential I-V characteristic can be described by formulae

$$\begin{aligned} E(x,t) &= E_a(t)(1-x^2/x_0^2),\\ B(x,t) &= B_{0,i} - \mu_0 J_c x - \mu_0 J_\delta x \ln\frac{E_a(t)}{E_C}\\ &\quad - \mu_0 J_\delta x\left[\left(1+\frac{x}{x_0}\right)\ln\left(1+\frac{x}{x_0}\right)\right.\\ &\quad \left. - \left(1-\frac{x}{x_0}\right)\ln\left(1-\frac{x}{x_0}\right) - 2\frac{x}{x_0}\right] \end{aligned} \tag{21}$$

where

$$E_a(t) = \frac{E_C\gamma_1^2}{2(1+2\delta)}\left(\frac{t_x}{t+t_0}\right)^{\frac{1}{1+2\delta}}.$$

The scaling approximation of the penetration depth position is given by

$$x_{a,0}(t) = \frac{B_{0,i}\gamma_1^{1+2\delta}}{\mu_0 J_C \delta}\left(\frac{E_C}{2(1+2\delta)E_a(t)}\right)^\delta \tag{22}$$

and according to equation (15) it will be increased as

$$x_0(t) = \frac{B_{0,i}}{\mu_0 J_C\left[1+\delta\ln\frac{E_a(t)}{E_C} + 2\delta(\ln 2 - 1)\right]}. \tag{23}$$

The constant t_0 can be determined using the electric field value $E_{0,i}$ on the surface of the superconductor at $t = t_i$. Then $t_0 = t_e - t_i$ where the characteristic time decay can be written as

$$t_e = \frac{B_{0,i}^2}{\mu_0 J_\delta E_C}\left(\frac{\gamma_1^2}{2(1+2\delta)}\frac{E_C}{E_{0,i}}\right)^{1+2\delta}.$$

The given formulae enable one to calculate the magnetic moment relaxation. For a superconducting slab of half thickness a it will be described by expression

$$\begin{aligned} -\frac{\mu_0 M(t)}{B_{0,i}} &= 1 - \frac{x_0(t)}{2a} + \frac{x_0^2(t)}{2a^2}\frac{B_p}{B_{0,i}}\\ &\quad + \delta\frac{x_0^2(t)}{4a^2}\frac{B_p}{B_{0,i}}\left[\ln\frac{E_a(t)}{E_C} + \frac{4}{3}\ln 2 - \frac{10}{9}\right] \end{aligned} \tag{24}$$

for all $x_0(t) < a$. Here, $B_p = \mu_0 a J_C$ is the field of complete penetration.

It is seen that the magnetic moment relaxation rate is practically proportional to the change in the moving boundary rate $\mathrm{d}x_0/\mathrm{d}t$. Moreover, similarly to the superconductors with the power I-V characteristic, the qualitative peculiarities of magnetic relaxation in superconductors with the exponential I-V characteristic are fully determined by the dynamics of the electric field relaxation on the surface of the superconductor.

Figure 6 shows the time-variation of the magnetization depth and the electric field on the surface of the low-temperature superconductor ($E_C = 10^{-4}$ V/m, $J_C = 4\times10^9$ A/m^2, $J_\delta = 4\times10^7$ A/m^2). The rate of the external magnetic field was set as $\mathrm{d}B/\mathrm{d}t = 1$ T/s and its increase was stopped at $t \geq 2\times10^{-4}$ s. In Figure 7 the results of numerical and analytical calculations are compared for the high-temperature superconductor with parameters $E_C = 10^{-8}$ V/cm, $J_C = 1.15\times10^5$ A/cm^2, $J_\delta = 8.686\times10^3$ A/cm^2. It was supposed that the external magnetic field increases at the rate $\mathrm{d}B/\mathrm{d}t = 7.5\times10^{-3}$ T/s and fixed at $t \geq 2\times10^{-4}$ s. As it has been mentioned above, the corresponding electric field distributions $E_0(x)$ specified at $t = t_i$ were taken according to the numerical simulation of the ramp rate stage.

The given solution indicates that, unlike superconductors with the power I-V characteristic, the relaxation in superconductors with the exponential I-V characteristic will be characterized by existence of three regimes. They are due to the corresponding variation of the electric field on the surface. Firstly, at $t - t_i \ll t_e$ the initial stage of the relaxation takes place. Secondly, the magnetic relaxation has the stage that can be called as quasi-scaling stage. Its duration depends on the superconductor's properties. For example, for a low-temperature superconductor the time of existence of its quasi-scaling stage is much longer than that for a high-temperature superconductor. During both stages the moving boundary of the magnetization region increases practically in accordance with the scaling law. This enables the scaling approximation to be used and the characteristic rates of the magnetic relaxation to be

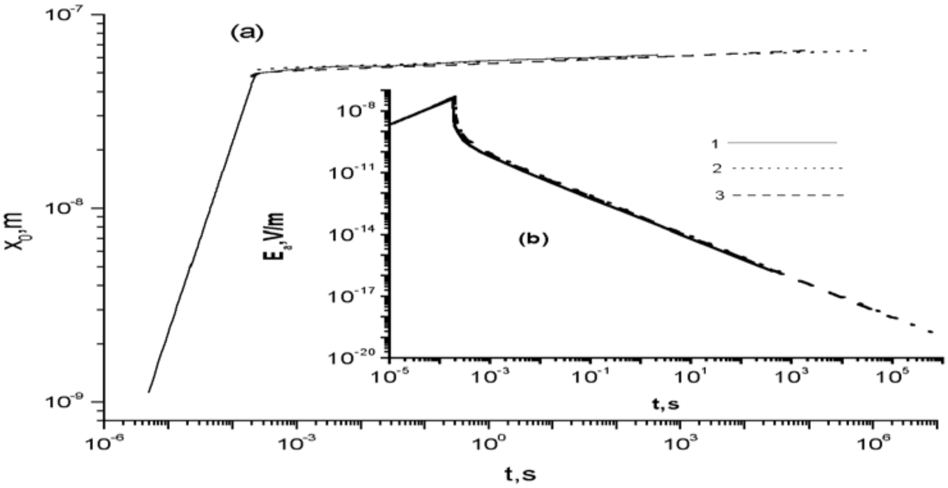

Fig. 6. Dynamics of moving boundary (a) and electric field (b) on the surface of Nb-Ti superconductor with the exponential I-V characteristic: 1 – numerical solution for the ramp rate and relaxation stages, 2 – scaling solution described by formulae (19, 21, 22), 3 – solution described by formulae (20, 21, 23).

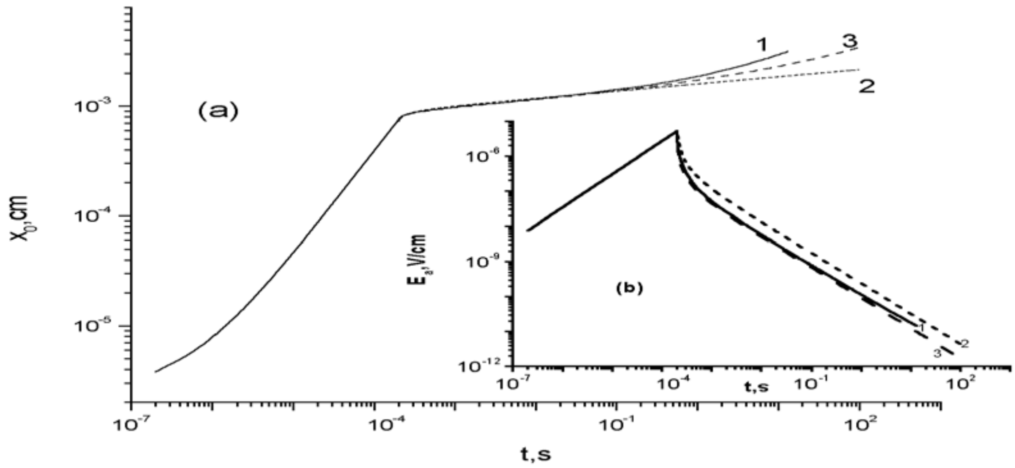

Fig. 7. Dynamics of moving boundary (a) and electric field (b) on the surface of YBCO superconductor with the exponential I-V characteristic: 1 – numerical solution, 2 – scaling solution described by formulae (19, 21, 22), 3 – solution described by formulae (20, 21, 23).

determined. For the initial stage it is nearly constant and can be estimated as

$$\frac{\mathrm{d}M}{\mathrm{d}t} \sim \frac{\mathrm{d}x_{a,0}}{\mathrm{d}t} \approx \frac{B_{0,i}\gamma_1}{\mu_0 J_\delta}\frac{\delta}{1+2\delta}\frac{1}{t_e}\left(\frac{t_e}{t_x}\right)^{\frac{\delta}{1+2\delta}}, \quad t - t_i < t_e.$$

The quasi-scaling stage is characterized by the rate decreasing with time and at $t - t_i > t_e$ it equals

$$\frac{\mathrm{d}M}{\mathrm{d}t} \sim \frac{\mathrm{d}x_{a,0}}{\mathrm{d}t} \approx \frac{B_{0,i}\gamma_1}{\mu_0 J_\delta}\frac{\delta}{1+2\delta}\frac{1}{t-t_i}\left(\frac{t-t_i}{t_x}\right)^{\frac{\delta}{1+2\delta}}.$$

Finally, at the last relaxation stage its evolution according to (15) lies outside the scaling law. This stage will be observed at the times, which can be estimated from inequality $x_0(t)/x_{a,0}(t) > 1$.

It is necessary to note the difference in the results of numerical and analytical calculations of the magnetization region boundary. As follows from Figure 7a , the approximate analytical calculations of $x_0(t)$ at the initial and quasi-scaling stages enable these stages to be described exactly enough. However, then the results of numerical and analytical calculations begin to differ. The numerical analysis of the corresponding electrodynamic states reveals the following regularity. The relaxation of the electromagnetic field induced in a superconductor with the exponential I-V characteristic is characterized by the formation of a relatively long conjugation region between the disturbed and undisturbed values of the electromagnetic field. Its size increases with time but the values of electric and magnetic fields induced inside this area are negligibly small. This end-effect is characteristic of both low and high-temperature superconductors. In the former case it is less significant. Therefore, the approximate solution formulated without taking into account the end-effect allows the electromagnetic field relaxation in

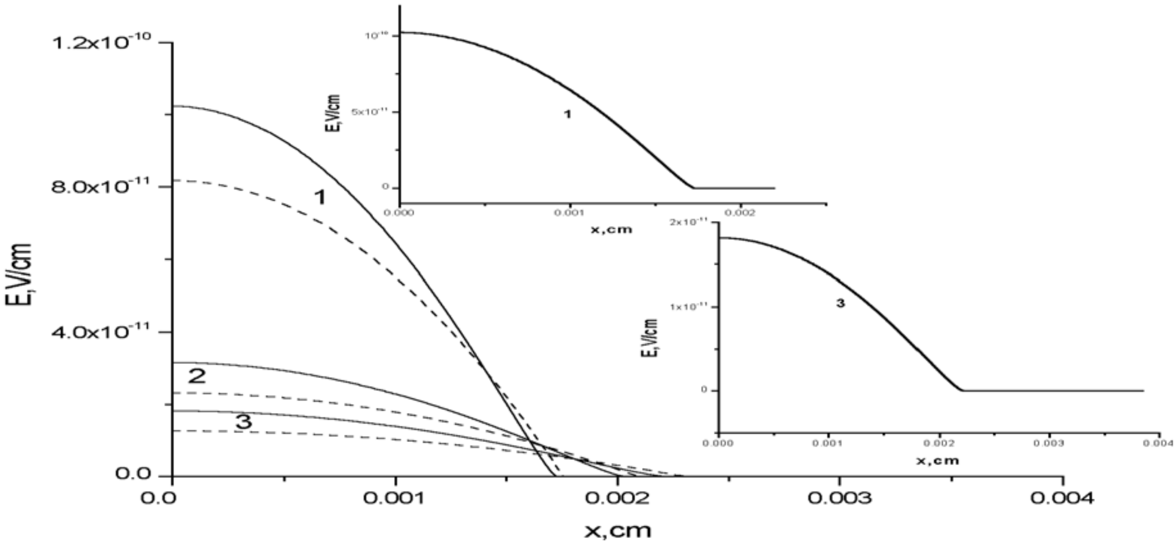

Fig. 8. Electric field distribution in YBCO superconductor with the exponential I-V characteristic determined according to the numerical solution (—) and solution described by formulae (20, 21, 23) (- - - -): 1 – $t = 1$ s, 2 – $t = 5$ s, 3 – $t = 10$ s.

low-temperature superconductors to be accurately described. However, it may lead to the discrepancy in the calculation of $x_0(t)$ for high-temperature superconductors. Figure 8 shows the corresponding results of numerical and analytical calculations of the spatial electric field distribution in a high-temperature superconductor during its relaxation. The curves in the insets of Figure 8 demonstrate the discussed peculiarities of conjugation region formation. A similar analysis was performed for superconductors with the power I-V characteristic. It reveals that, unlike the states of the superconductors with the exponential I-V characteristic, the end-effect of the conjugation region in superconductors with the power I-V characteristic is practically absent.

6 Conclusions

The above theoretical results indicate the existence of the magnetic relaxation peculiarities in superconductors with the power and exponential I-V characteristics describing the flux creep in the macroscopic approximation. It is shown that in the flux-creep regime a special state is established in response to arbitrary external perturbations. As a result, the relaxation phenomena are characterized by the formation of electromagnetic wave propagating in the superconductor at a finite rate. In this case electric field and magnetic induction induced inside a superconductor by external perturbations smoothly approach their undisturbed values at moving penetration boundary.

A comparative analysis of the relaxation dynamics in superconductors with the power and exponential I-V characteristics shows that the equivalence of the obtained results depends on the character of the power I-V characteristic. Considerable difference will be seen at $n < 10$ when the creep strongly affects the electromagnetic field distribution. The proposed analytical solutions also show that the main qualitative difference is in the dynamics of the electric field on the surface of the superconductor, which determines the character of the magnetic relaxation. For a superconductor with the power I-V characteristic it is described by two characteristic regimes with the rates depending on the superconductor's properties and the history of the external magnetic field change. At the same time, relaxation phenomena in superconductors with the exponential I-V characteristic have three stages. The first two stages are qualitatively analogous to the stages of a superconductor with the power I-V characteristic. But at the final stage of relaxation its rate is higher than the rate of relaxation in a superconductor with the power I-V characteristic. As a result, the magnetic relaxation in a high-temperature superconductor with the exponential I-V characteristic is more intensive than that in a superconductor with the power I-V characteristic.

These peculiarities are connected with the different character of the change of the power and exponential I-V characteristics in the low electric field area and thus correspond to the different variation of the differential resistivity of a superconductor in this electric field area. As follows from (1, 3), at the same value of induced screening current density the differential resistivity of the superconductor with the exponential I-V characteristic may be lower than that in the superconductor with the power I-V characteristic. Therefore, the electric field will decay more intensively into the superconductor with power I-V characteristic than into that with exponential I-V characteristic. As this difference is more noticeable in I-V characteristic of the high-temperature superconductors, the qualitative and quantitative difference in the magnetic relaxation occurring in the superconductors with the exponential and power I-V characteristics will be the most noticeable in high-temperature superconductors. These results should be taken into consideration when equations (1, 3) will be used to describe experimental data of I-V characteristics and especially for high-temperature superconductors.

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